## AP Calculus AB

Transformation Summary
Dr. Paul L. Bailey
September 4, 2017

Given a subset of $\mathbb{R}^{2}$, we may modify the position or shape of the set by transforming each of the coordinates in each point of the set, according to the chart below.

| Shift right $h$ units | $x \mapsto x+h$ |
| :--- | :---: |
| Shift left $h$ units | $x \mapsto x-h$ |
| Shift up $k$ units | $y \mapsto y+k$ |
| Shift down $k$ units | $y \mapsto y-k$ |
| Stretch horizontally by a factor of $a$ | $x \mapsto a x$ |
| Shrink horizontally by a factor of $a$ | $x \mapsto \frac{x}{a}$ |
| Stretch vertically by a factor of $b$ | $y \mapsto b y$ |
| Shrink vertically by a factor of $b$ | $y \mapsto \frac{y}{b}$ |
| Reflect across the $y$-axis | $x \mapsto-x$ |
| Reflect across the $x$-axis | $y \mapsto-y$ |

Transformations of Sets

Given an equation, we may transform the locus of the equation by modifying the original equations according to the rules laid out in the chart below.

| Shift right $h$ units | $x \mapsto x-h$ |
| :--- | :---: |
| Shift left $h$ units | $x \mapsto x+h$ |
| Shift up $k$ units | $y \mapsto y-k$ |
| Shift down $k$ units | $y \mapsto y+k$ |
| Stretch horizontally by a factor of $a$ | $x \mapsto \frac{x}{a}$ |
| Shrink horizontally by a factor of $a$ | $x \mapsto a x$ |
| Stretch vertically by a factor of $b$ | $y \mapsto \frac{y}{b}$ |
| Shrink vertically by a factor of $b$ | $y \mapsto b y$ |
| Reflect across the $y$-axis | $x \mapsto-x$ |
| Reflect across the $x$-axis | $y \mapsto-y$ |

Given a function $f$, we consider the equation $y=f(x)$ and obtain its locus. We may transform the graph of this locus by modifying the function according to the chart below.

| Shift right $h$ units | $f(x-h)$ |
| :--- | :---: |
| Shift left $h$ units | $f(x+h)$ |
| Shift up $k$ units | $f(x)+k$ |
| Shift down $k$ units | $f(x)-k$ |
| Stretch horizontally by a factor of $a$ | $f\left(\frac{x}{a}\right)$ |
| Shrink horizontally by a factor of $a$ | $f(a x)$ |
| Stretch vertically by a factor of $b$ | $b f(x)$ |
| Shrink vertically by a factor of $b$ | $\frac{f(x)}{b}$ |
| Reflect across the $y$-axis | $f(-x)$ |
| Reflect across the $x$-axis | $-f(x)$ |
| Transformations of Functions $f(x)$ |  |

